may exert a considerable influence on the dynamic of crack propagation.

REFERENCES

1. BARENBLATT G.I. and CHEREPANOV G.P., On the cleavage of brittle bodies, PMM, 24, 4, 1960.
2. BARENBLATT G.I., On some problems of the theory of elasticity arising in the study of the mechanism of hydraulic fracture of an oil-bearing stratum, PMM, 20, 4, 1956.
3. AFANAS'YEV E.F. and CHEREPANOV G.P., Some dynamic problems of the theory of elasticity, PMM, 37, 4, 1973.
4. SHER E.N., On the stress state of an isolated rectilinear cut growing at a constant rate and acted upon by concentrated forces from inside, PMTF, 1, 1980.
5. GORDEYEV YU.N. and KUDRYASHOV N.A., Dynamics of normal separation crack growth during its cleavage by a gas flow, PMM, 52, 2, 1988.
6. GORDEYEV YU.N., Selfsimilar problems of star-shaped crack development under the action of a cleaving gas flow, PMM, 53, 2, 1989.
7. BELYAYEV B.M., VASIL'YEV S.A. and NIKOLAYEV S.I., Fracture of a Stratum by Powder Gas Pressure, VNIIOENG, Moscow, 1967.
8. MIKHALYUK A.V., Torpedoing and Impulsive Hydraulic Fracturing of Strata. Naukova Dumka, Kiev, 1986.
9. KOSTROV B.V., Mechanics of the Focus of a Tectonic Earthquake, Nauka, Moscow, 1975.
10. ROZHDESTVENSKII B.L. and YANENKO N.N., Systems of Quasilinear Equations and their Applications in Gas Dynamics, Nauka, Moscow, 1978.

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# THE EQUATIONS OF MOTION OF CONDENSED MEDIA WITH CONTINUALLY KINETIC FRACTURE*

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A continual model of fracture /1, 2/ within the framework of which the degree of damage to the material is determined by the volume of the micropores or voids formed as a result of increasing tensile stresses, is reformulated to cover the case of viscoelastic media with finite deformations. As a result, equations of motion of a viscoelastic medium with continual fracture are proposed. In the case of a medium without fracture the equations are identical with the equations of motion /3-6/, and when the damage is small and the loading uniaxial, they reduce to the well-known equations /1/. The properties of certain simplest flows are studied using the model proposed.

A large volume of literature exists, dealing with the rheological models of a continuous condensed medium, describing strength effects. A phenomenological approach to constructing the defining relations, including, in the limit, the hydrodynamic as well as elastic modes of motion of the material, which retains its contnuity, is given in /3, 7/. The problem of including fracture in such models has received less attention. A survey is given in /4/ of work done up till now dealing with this problem, and a theory of the continual fracture of non-linearly elastic model based on a phenomenological approach is developed. A second rank tensor whose properties were studied in /4/ is used as the macroscopic measure of material damage. By virtue of the assumptions made in /4/, it is established that the increase in the damage in thermo-elastic media is governed not by the kinetic equation, but by a finite

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relation connecting the extent of the damage with the value of the deformation, entropy and the distributed source of the damage. Existing experimental data show that, for example, the cleavage in condensed material can satisfactorily described using the continual kinetic model of fracture, where the volume of the voids or pores formed serves as a measure of the damage /1/. Since this model has an obvious physical meaning, it is found to be useful for modelling cleavage phenomena since it describes, in a continuous manner, the transition from a continuous material to a marocrack with subsequent separation of the split plate. This is particularly attractive when the computations are carried out using the through-count scheme.

We shall assume that the distintegrating medium can be represented in the form of a matrix of continuous condensed material containing micropores. Let us consider the wave dynamics of such a medium. We introduce the following macroscopic variables: $V_p$ is the volume of the void per unit volume, $\rho^\circ$ is the natural density of condensed material and $\rho$ is its averaged density $\rho = \rho^\circ(1 - V_p)$. Henceforth, we shall adhere to the model of mutually interpenetrating continua /8, 9/ and carry out our investigation in a Cartesian system of coordinates with basis vectors $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$.

We have the following expressions in the integral laws of conservation of mass, momentum and energy:

$$\frac{d}{dt}\int\limits_{\omega}\rho^\circ\,dV_M = -\int \rho^\circ\mathbf{u}\mathbf{n}\,dS,\quad \frac{d}{dt}\int\limits_{\omega}\rho^\circ\,dV_M = \int \boldsymbol{\sigma}\cdot\mathbf{n}\,dS$$
$$\frac{d}{dt}\int\limits_{\omega}\rho^\circ e_T\,dV_M = \int\limits_{S}\boldsymbol{\sigma}:\mathbf{n}\mathbf{u}\,dS,\quad e_T = e + \frac{1}{2}u^2$$

($e_T$ and $e$ are the total specific and internal energy, $\boldsymbol{\sigma} = \|\sigma_{ik}\|$ is the stress tensor and $\mathbf{u} = (u_1, u_2, u_3)$ is the velocity vector). The volume integrals are taken over the volume occupied by the continuous medium.

The surface integrals must be studied in more detail. By virtue of the previous assumption concerning the structure of the material, the boundary of the element of the continuous medium $S$ represents the union of the outer boundary $S_e$ and the boundaries of the micropores $S_i$, lying within the volume $\omega$. The surface integrals in the equations of motion are calculated over the union of $S_e$, $S_i$.

We have, by the definition of $V_l, \rho, \rho^\circ, \rho^\circ dV_M = \rho dV$, and the volume integrals reduce to integrals over the volume bounded by $S_e$.

Let us, for example, evaluate the integral on the right-hand side of the equations of conservation of momentum. Using Gauss's theorem we have

$$\int\limits_{S}\boldsymbol{\sigma}\mathbf{n}\,dS = \int\limits_{\omega}\nabla\boldsymbol{\sigma}\,dV - \int\limits_{\omega}\nabla\boldsymbol{\sigma}V_p\,dV$$

On changing from surface to volume integrals, we assumed that $\sigma$ can be continued smoothly into the micropores. This assumption needs special justification. For example, in the hydrodynamic approximation the situation in which $p = 0$ at the boundary with the vacuum is impossible. It makes sense to speak only of some mean pressure over a physically infinitesimal volume of the porous medium, and also assign it to the free surfaces, which is only possible at a low concentration of pores: $V_p \ll 1$. On the other hand, it is clear that the equations can also be used in the limiting situation when the material has disintegrated and the stresses within it are zero. In this sense the equations are uniformly suitable.

Transforming the surface integrals in the integral laws of conservation into volume integrals in the standard manner, we obtain

$$\partial\rho/\partial t + \nabla\rho\mathbf{u} = 0,\ \partial\rho\mathbf{u}/\partial t + \nabla(\rho\mathbf{u}\otimes\mathbf{u}) = (1 - V_p)\nabla\boldsymbol{\sigma}\mathbf{u},$$
$$\partial\rho e_T/\partial t + \nabla(\rho\mathbf{u}e_T) = (1 - V_p)\nabla\boldsymbol{\sigma}\mathbf{u} \qquad (1)$$

We write the equations of compatibility of velocities are deformations, just as in the case of a continuous medium /6/ ($\mathbf{A}$ is the total distortion tensor)

$$\partial\mathbf{A}/\partial t + (\mathbf{u}\nabla)\mathbf{A} = \mathbf{U}\mathbf{A};\ \mathbf{A} = \|A_{ik}\| = \|\partial x_i/\partial x_k\|,\ \mathbf{U} = \|U_{ik}\| = \|\partial u_i/\partial x_k\| \qquad (2)$$

The Lagrangian coordinates $x_k$ $(i, k = 1, 2, 3)$ represent the coordinates of an element of the continuous medium in its initial configuration.

The equations of motion should be closed with the defining relations for the stress tensor $\boldsymbol{\sigma}$ and pore volume $V_p$.

In the case of hyperelastoplastic media it was suggested in /8/ that $\boldsymbol{\sigma}$ should be determined in terms of the characteristics of a continuous medium. We shall use this approach below. First we consider the hyperelastic medium. In accordance with what was said above, we assume that the specific internal energy is a function of the entropy $s$ and the tensor $\mathbf{A}^\circ = (\rho/\rho)^{1/3}\mathbf{A}$, and, that the first law of thermodynamics holds

$$de = Tds + \rho^{\circ-1}\boldsymbol{\sigma}: d\mathbf{A}^\circ\mathbf{A}^{\circ-1} \tag{3}$$

($T$ is the temperature). The above equation and an expression for the stress tensor $\sigma = \rho^\circ (\partial e/\partial\mathbf{A}^\circ)_s\mathbf{A}^{\circ *}$ which follows from it, are identical with the corresponding relations for the continuous hyperelastic medium /5, 6/, with the total distortion tensor $\mathbf{A}$ replaced by the tensor $\mathbf{A}^\circ$ introduced earlier. Of course, the choice of defining relations is not unique and corresponds to the choice of damageability in the form of a scalar /4/. The relation $\rho^\circ \det \mathbf{A}^\circ = \rho_0$, where $\rho_0$ is the density of the medium in the initial configuration, holds by virtue of the properties of the total distortion tensor.

The volume of the voids $V_p$ increases as new pores are formed, and also by virtue of the motion of the material, which reflects the law of pore growth within a volume $\omega$: $\partial V_t/\partial t + \mathbf{u}\nabla V_p = \psi$. The following divergent form corresponds to this equation in the regions of smoothness by virtue of the equations of continuity:

$$\partial\rho V_r/\partial t + \nabla\rho\mathbf{u}V_p = \rho\psi \tag{4}$$

From this it follows, in particular, that the volume $V_p$ does not change. This condition has a clear physical meaning, since under an instantaneous compression of the pores are frozen into the medium and their volume decreases in exactly the same way as the volume of a continuous medium.

Thus the divergent form (4) retains its validity in the case of discontinuous solutions. Expressing the volume $V_p$ in terms of the specific volume $V_p'$ of the pores $V_p = \rho V_p'$, introduced in /1/, we obtain an equation which becomes, when $Vp' \ll 1$, an equation /1/ for the kinetics of pore growth when $\psi/\rho = V_T^{\cdot}$.

We obtain the following expression for the increase in entropy from Eqs.(1) and (2):

$$Tds/dt = -p\psi/\rho \quad (p = -{}^1/_3 \operatorname{tr} \boldsymbol{\sigma})$$

($p$ is the hydrostatic pressure). The condition of non-decrease in entropy imposes a restriction on the form of the functions $\psi$ in the phenomenological approach used here, namely that the inequality $-p\psi \geqslant 0$ must hold.

This condition holds for the function specifying the kinetics of pore growth /1/, since the pores increase in size only under tensile stresses. In fact, the result obtained only states that the model used here does not contradict the second law of thermodynamics. It is entirely possible that the energy of fracture calculated using this model will not agree with the experimental results. The situation can be corrected by introducing into the energy equation terms /4/ connected with the formation and change of the surfaces of the microcrack edges and the number of microdefects.

The equations of motion of a porous medium are, with the exception of the equation of continuity, non-divergent. Nevertheless, we can write for them the relations at the discontinuity in a standard manner /6/ since, as follows from the derivation of the equations, the terms $V_p\nabla\boldsymbol{\sigma}$ and $V_p\nabla\boldsymbol{\sigma}\mathbf{u}$ represent volume sources which drop out when the integration is carried out over an infinitesimally small volume. With regard to the equations of compatibility of deformations, they can be reduced /5, 6/ to divergent form and the conditions at the discontinuity for them can be reduced to the requirement that the displacements must be continuous. The relations at the discontinuity are written, in a system of coordinates attached to the shock wave, in the form /6/:

$$[\rho A_{1l}] = 0,\ -j\,[A_{kl}] = [u_k]\rho A_{1l}{}^1,\ -j\,[u_i] = [\sigma_{i1}],\ [e] = (\sigma_{i1}{}^1 + \sigma_{i1}{}^2)[A_{i1}]/(2\rho A_{11});\quad l, i = 1, 2, 3;\ k = 2, 3;\ [f] = f^1 - f^2$$

Here the direction of the $x_1$ axis coincides with the normal to the shock wave, the superscripts 1 and 2 denote the parameters behind and in front of the discontinuity, and $j$ is the flux of material across the discontinuity.

The relations given above are identical with the corresponding expressions for a continuous medium, with the sole difference that the averaged density $\rho$ is used here. The condition at the discontinuity for the volume of the pores $[V_p] = 0$ is added to them.

In the limit of fairly strong shock waves and in the hydrodynamic approximation, the relations at the discontinuity become the relations for the porous medium /1/, provided that we assume that a continuous material $V_p{}^1 = 0$ is present behind the discontinuity. Within the framework of the model we have used, the stationary structure consisting of the bow shock where the pore volume does not change, and the adjacent region in which the pores are closed up in accordance with the function $\psi$ correspond to this transition. Since $-p\psi > 0$, the second law of thermodynamics also holds.

Let us investigate the properties of some flows using the model used. We shall assume for simplicity that the hydrodynamic approximation holds. It will become clear later that the passage to the general case gives rise to no difficulties.

We shall study the structure of the shock wave in the short-wave approximation /11/. Using standard techniques, we arrive at the following equations for $u_1$, $V_p$:

$$2\left(\frac{\partial u_1}{\partial t}+m_0u_1\frac{\partial u_1}{\partial \xi}\right)=-a_0^2\frac{\partial V_p}{\partial \xi},\quad a_0\frac{\partial V_p}{\partial \xi}=-\psi$$

$$\xi=x_1-a_0t,\quad m_0=\frac{1}{2a_0^2\rho_0^3}\left(\frac{\partial^2 p_0}{\partial V^2}\right)_s,\quad a_0^2=\left(\frac{\partial p_0}{\partial \rho}\right)_s,\quad p=\rho_0a_0u$$

We can assume that $\psi=-V_p/\tau$ when $p>0$. Such a choice ensures that the pores close up under compression. A stationary wave with velocity $D$ obeys the following system of ordinary differential equations:

$$2\left[m_0u_1-(D-a_0)\right]\frac{du_1}{d\xi}=-a_0^2\frac{dV_p}{\partial \xi},\quad a_0\frac{\partial V_p}{\partial \xi}=-\psi$$

If the porosity of the material in front of the wave is equal to $V_0$, integration of the first equation will yield

$$D={}^1/_2m_0u_p-{}^1/_2a_0^2V_0/u_p+a_0$$

where $u_p$ and $u_s=2D/m_0$ are the velocities at $-\infty$ and at the front.

The above solution becomes invalid when $u_p<\sqrt{V_0/m_0}$, since $D<a_0$, $u_s<0$ and the wave is no longer evolutionary. In this case the structure of the flow is as follows: in the precursor situated in front of the wave, with an amplitude decaying with time, the material compacts under zero pressure to porosity $V_0'=m_0u_p^2/a_0^2<V_0$, and then compresses in a continuous stationary structure governed by the formulas given above with $D=a_0$, and porosity before the front of $V_0'$. This statement is confirmed by numerical solutions of the non-stationary wave equations using the method of characteristics.

Let us inspect the structure of the flow within the zone of fracture with cleavage. Clearly, during the late stage of fracture we must assume that the natural density changes weakly with time in every particle. Retaining the principlal terms in the equation of continuity, we have

$$\rho^\circ\partial u_1/dx_1=\rho^\circ(1-V_p)^{-1}\psi=\Phi_1 \tag{5}$$

We shall assume that the flow is isentropic, which is true for sufficiently weak shock waves. Expressing the pressure in terms of the density, we rewrite the momentum equation in the form

$$\partial u_1/\partial t+u_1\partial u_1/\partial x_1=-(a_0^2/\rho^\circ)\,\partial\rho^\circ/\partial x_1 \tag{6}$$

The quantity $\Phi_1$ can obviously be written, under the assumptions made, in the form

$$\Phi_1=(\rho_0-\rho^\circ)/\tau \tag{7}$$

The last formula reflects the tendency of the natural density to approach its own value when the pressure is zero. The qualitative regularities of the flow in the zone of disintegration can be exhaustively studied assuming that the time of disintegration $\tau$ is constant. In this case, eliminating $\rho^\circ$ from (5)-(7) we arrive at the equation

$$\partial u_1/\partial t+u_1\partial u_1/\partial x_1=a_0^2\tau\partial^2u_1/\partial x_1^2$$

This equation is identified with the Burgers equation which describes wave motions of a viscous, heat conducting gas /11/. We assume that the size of the fracture zone is much smaller than the length of the impulse. Then the boundary conditions will state that $u_1=\overset{\circ}{u}_{1,2}$ as $x_1\to\pm\infty$, where $\overset{\circ}{u}_{1,2}$ are the velocities of the free surfaces after separation of the detached plate.

We will choose, as the solution of above problem, the solution of the Cauchy problem with initial conditions

$$u_1|_{t=0}=\begin{cases}u_1^\circ, & x_1>0\\ u_2^\circ, & x_1<0\end{cases}$$

The solution is given in /11/, and at long time intervals it has an asymptotic form which is identical with the solution of the problem of a centred rarefaction wave in an ideal gas. The solution has a clear physical meaning. The particles of fractured material move along rectilinear trajectories at constant velocities. A detailed analysis shows that the flow transfers into the selfsimilar mode in accordance with the rule $t^{-1/2}$. In the neighbourhood of free surfaces $u_1=\overset{\circ}{u}_{1,2}+O(e^{-\beta t})$, i.e. they are fairly sharply localized.

The problem of localizing the detached plate can be discussed from another viewpoint. Let us find the mass of the fractured material as $t \to \infty$:

$$M = \int\limits_{u_2^\circ t}^{u_1^\circ t} \rho^\circ (1 - V_p)\, dx_1$$

Here the integration is carried out for a constant value of time. With $u_1$, we have the equation $dV_p/dt = (1 - V_p) / t,\ t \to \infty$, for the pore volume, in which the differentiation is carried out for a constant value of $x_1/t$. This yields

$$V_p = 1 - C(\xi) / t,\ \xi = x_1/t$$

We note that the above analysis does not describe the initial phase of the disintegration where the assumptions made are invalid. The value of $C(\xi)$ must be found by matching with the solution at the initial phase of the process of duration $\tau$. Finally we have

$$M = \rho_0 \int\limits_{u_2^\circ}^{u_1^\circ} C(\xi)\, d\xi$$

From this we conclude that $M$ tends to a constant value with time and this, in turn, indicates the local nature of the fracture process. We note that the flow is essentially non-linear. In the linear approximation the disintegration is not localized.

Let us consider, in the hydrodynamic approximation, the situation limited to a known degree for the fracture model described above. We shall assume that when the tensile stresses acting on the particles of the medium exceed the cleavage values, the material fractures and ceases to resist separation. We shall describe the medium as a powder with zero pressure within the particles. If the density of the continuous material is equal, at the instant preceding the disintegration, to $\rho_1$, then after disintegration, this value will be assigned to the average density. In order to determine the true density of disintegrated material we should employ some physical model. We can, for example, assume that the process is isentropic, or take into account the energy of disintegration.

The motion of the power is described by a system of equations for a medium without pressure which has, in the one-dimensional case /12/, a triply degenerate characteristic $dx_1/dt = u_1$, but is not hyperbolic, since it does not reduce to its characteristic form. The discontinuities in such a medium were studied using the model of rigid, non-interacting particles in /12/. In particular, if the homogeneous half-flows of the particles occupying the half-spaces $x_1 > 0$ and $x_1 < 0$, move in different directions, a particle-free region will form within the flow, while if they are directed against each other, the particle trajectories will intersect and the medium will become a two-velocity medium. Kraiko /12/ tried to eliminate such an ambiguity by introducing an impermeable surface of discontinuity possessing a finite mass, momentum and energy.

In the last case it seems reasonable to adopt, for the disintegrated concentrated material, a scheme of flow is which the shock waves move over both states and the relations for a porous shock adiabatic /10/ hold on the wave. The material is continuous behind the shock wave, and the states behind the shock waves combine in the usual manner by virtue of the condition of continuity at the contact discontinuity of the pressure and velocity. The structure of the shock waves is disregarded when using such an approach.

We can construct a scheme of disintegration of any discontinuity in which the disintegrated material lies adjacent to the continuous material. If the rate of unloading of the continuous material into a vacuum is greater than the velocity of the powder, there is no interaction. The powder moves inertially with constant velocity. In the opposite case a shock wave moves into the disintegrated material, and a shock or rarefaction wave into the continuous material.

With slight modifiations the above discussion can be applied to a hyperelastic medium with a fracture.

Let us now consider the problem of the model of a disintegrating viscoelastic medium. Within the framework of the relaxation model /6/ the total distortion tensor $\mathbf{A}$ can be represented in the form $\mathbf{A} = \mathbf{A}_e\mathbf{A}_p$ where $\mathbf{A}_e$ and $\mathbf{A}_p$ are the elastic and plastic distortion tensors /5, 6/, the kinetic equation for the plastic deformation $\mathbf{A}_p^{\cdot} = \mathbf{\Phi}(\mathbf{A}_e, \mathbf{A}_p, s)$ holds, and the internal energy is $e = e(\mathbf{A}_e, s)$. Let us write the distortion tensor in the form

$$\mathbf{A} = \mathbf{A}_e^\circ \mathbf{A}_p^\circ,\ \mathbf{A}_e^\circ = (\rho/\rho^\circ)^{1/3} \mathbf{A}_e,\ \mathbf{A}_p^\circ = (\rho^\circ/\rho)^{1/3} \mathbf{A}_p$$

and assume that $e = e(\mathbf{A}_e^\circ, s)$. Now we have the following kinematic equation for $\mathbf{A}_p^\circ$:

$$\mathbf{A}_p^{\circ\cdot} = (1 - V_p)^{-1/3}\mathbf{\Phi} + {}^1/_3 (1 - V_p)^{-1} \mathbf{A}_p^\circ V_p^{\cdot} \tag{8}$$

This, together with equations of motion, yields the following expression for the increase in entropy

$$Tds = (1/\rho^\circ)\,\sigma : \mathbf{A}_e{}^\circ \Phi \mathbf{A}_p^{\circ-1} \mathbf{A}_e^{\circ-1} (1 - V_p)^{-1/3} - (1/\rho^\circ)\, p V_p{}^\cdot$$

The first term on the right-hand side is positive by virtue of the demands imposed on $\Phi$, and the second term by virtue of the reasons already listed. Thus the second law of thermodynamics retains its validity for the model of a viscoelastic fracturing medium. Relations (1), (2), (4) and (8) form a complete system of equations describing the motion of a viscoelastic medium with fracture.

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REFERENCES

1. KANEL G.I., RAZORENOV S.V. and FORTOV V.E., Kinetics of fracture of AMg6M alluminium alloy with cleavage. PMTF, 5, 1984.
2. BARBEE T.W., SEAMAN L., CREWDSON R. and CURRAN D., Dynamic Fracture Criteria for Ductile and Brittle Metals. J. Mater. 7, 3, 1972.
3. GODUNOV S.K., Elements of the Mechanics of a Continuous Medium, Nauka, Moscow, 1978.
4. KONDAUROV V.I.,Continual fracture of non-linearly elastic bodies. PMM, 52, 2, 1988.
5. KONDAUROV V.I., On the equations of an elastoviscoplastic medium with finite deformations. PMTF, 4, 1982.
6. NI A.L. and FORTOV V.E., Divergent system of non-stationary equations of motion of visco-elastic media in Euler coordinates, PMM, 51, 6, 1987.
7. GODUNOV S.K. and ROMENSKII E.I., Non-stationary equations of the non-linear theory of elasticity in Eulerian coordinates. PMTF, 6, 1972.
8. RAKHMATULLIN KH.A., Fundamentals of gas dynamics of interpenetrating motions of compressible media. PMM, 20, 2, 1956.
9. KRIKO A.N. and STERNIN L.E., On the theory of flows of a two-velocity continuous medium with solid or liquid particles. PMM, 29, 3, 1965.
10. ZEL'DOVICH YA.B. and RAIZER YU.P., Physics of Shock Waves and High Temperature Hydro-dynamic Phenomena, Nauka, Moscow, 1966.
11. RYZHOV O.S., Non-linear acoustics of chemically active media. PMM, 35, 6, 1971.
12. KRAIKO A.N., Discontinuity surfaces in a medium deprived of "natural" pressure. PMM, 43, 3, 1979.

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# BOUNDS ON CONTROL IN THE LINEAR DYNAMIC OPTIMIZATION PROBLEM WITH A QUADRATIC FUNCTIONAL*

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Some bounds of the region from whh a linear system can go in a prescribed time to the origin with a given value of an integral functional that is quadratic in control are derived. A bound on the required control is given. Conditions are proposed when a controller can be designed taking any point from a given bounded region to the origin in a prescribed time with control not exceeding the specified bound.

The design of programmed bounded controls that take a linear system in a finite time to a prescribed state is considered in /1, 2/.

Consider a linear controlled system

$$x^\cdot = Ax + bu \tag{1}$$

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